INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR

Date: …….*FN / AN* Time: *60 min* Full marks: 40 No. of students: 69

Autumn Semester Class Test 2, 2017 Dept: *Computer Science & Engineering* Sub No: CS60005

M.Tech (Core) Sub Name: ***Foundations of Computing Science***

Instructions: *Answer all questions*

---

1. For each of the following statements indicate whether the statement is true or false and give a reason (counterexample or proof) supporting your answer.
   (a) All problems in P are also in NP and in co-NP.
   (b) If languages L_1 and L_2 are in P, then the language $L_3 = L_1 \cap L_2$ is also in P.
   (c) Let problems $A \in P$, and $B \in$ NP-complete. If $A \leq_P B$, then A must be NP-Complete.
   (d) Let problems $A \in P$, and $B \in$ NP-complete. If $B \leq_P A$ then it is true that P = NP.
   (e) All problems in NP can be solved using a *deterministic* Turing machine in *polynomial space.*

   [5×3 = 15 marks]

2. For each of the following problems, state if it is known to be in **NP**, **co-NP**, or more precisely in **P.**
   (a) SAT: Given a Boolean formula in CNF, does the formula have a satisfying assignment of its variables?
   (b) VALIDITY: Given a Boolean formula in CNF, is the formula valid?
   (c) k-CUT: Does a given graph, G, have an edge cut of size less than k?
   (d) Non-VALIDITY: Given a Boolean formula in CNF, does the formula have any assignment of its variables that makes the formula *False*?
   (e) k-REGALLOC: Given a set of variables and time intervals in which each variable is in use ("*live*"), is there an allocation of variables to registers that uses less than k registers?

   [5×2 = 10 marks]

3. Consider the SET-COVER problem defined as follows:

   SET-COVER = { ⟨U,S,k⟩ | U is a finite set of numbers, S is a collection of sub-sets of U, there is a k-sized cover of U from the collection S}

   A cover $C \subseteq S$ is a collection of sub-sets whose union is U.
   (a) Prove that the SET-COVER problem is NP-Complete. Clearly indicate which problem is being reduced to which problem and clearly show the steps of the reduction, proving the reduction is in P. [**Hint**: Use the fact that VERTEX-COVER is NP-Complete]
   (b) Consider a variant of the SET-COVER problem: "Is the minimum sized set cover to cover all elements in U of size k?". What can you say about this problem?

   [10+5 = 15 marks]